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A glance into single molecules and stochastic processes

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by

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Abstract

This thesis is divided into two chapters. In chapter *1*, we deal with *a basic study of stochastic processes.* In chapter *2*, we *glance into single molecules.* The models and results obtained in chapter *1* when studying stochastic processes that take place on finite chains are used in chapter *2* for analyzing and explaining the dynamical behaviors of single molecules. Nevertheless, each of the chapters stands on its own. Particularly, the motivation for studying the statistical properties of stochastic processes in finite systems is also due to pure theoretical interest. On the other hand, the theory and models developed for analyzing the dynamical behaviors of single molecules go beyond the framework of stochastic processes in finite chains, and involve also relationships to random walks on complex networks, information theory, thermodynamics, polymer physics, and biological considerations.

Structurally, each of the chapters in the thesis is divided into four sections. Each chapter starts with a general introductory section followed by three sections that give the actual work. In each of these sections, we start with a specific introduction, in which we explain the basic terminology of the subject, describe the investigated system, and give the motivation for studying it. Then, we briefly list our related results. The full details are given in the papers that follow the specific introduction sections.

We shortly present the topics covered in this thesis. In chapter *1* section *1.2*, we study dynamics on finite one-dimensional Markov chains and use the result to model two types of processes involving dendrimers, which are synthetic tree-like shaped organic molecules. The first process is the migration of energy across dendrimers, where the corresponding application is the use of dendrimers as efficient light

harvesting antennae. The second process is the gradual disintegration of dendrimers into its basic subunits, where the corresponding application is the use of dendrimers as targeted drug-platforms. Our analysis establishes relationships between the structure of the dendrimer and the efficiency of the process occurring in it. In section *1.3*, we study dynamics in finite one-dimensional semi-Markov arbitrarily inhomogeneous chains. We derive closed-form analytical expressions for the propagators of the process in Laplace space of appropriate dimensions. These novel and general formulae fill a gap in the literature of one-dimensional nearest-neighbor semi-Markovian hopping processes (known also as continuous time random walks). Using these expressions, we solve a particular system that models the activity of RNA polymerase. We show that the polymerization time of a segment of length L can be dramatically changed from a linear to exponential scaling with L, depending on a single parameter that gives the probability of immature termination of polymerization. In section *1.4*, we study a resonant phenomenon that is obtained when a random walk is taking place on coupled one-dimensional finite chains. This system exhibits a resonant phenomenon in the mean exit time (and in other characteristics) at a particular value of the coupling rate depending on the details of the system. We explain why this phenomenon is obtained, and determine the bounds on the system characteristics that still enable observing the resonant phenomenon. Relationships between the resonant phenomenon and structural fluctuations during activity of biopolymers are also discussed. In chapter *2* section *2.2*, we introduce a theory and various statistical methods for analyzing two-state trajectories that are obtained from single molecule measurements. We use some of these methods in analyzing two-state trajectories obtained from experiments measuring the activity of single enzymes, which is presented in section *2.3*. [This is an outcome of collaboration between our

group and two groups of experimentalists, from Leuven (F. C. De Schryver group) and from Nijmegen (R. J. M. Nolte group)]. We show that the enzymatic activity is correlated and highly non-exponential. These observations are in contrast with the conventional description of enzymatic activity by the Michaelis-Menten model. Finally, in the last section of this thesis, section *2.4*, we theoretically study the translocation of biopolymers through individual nanopores. This is another example for a system being studied on the single molecule level that experimental output is a two-state trajectory. From the mathematical aspect, the same formalisms used in sections *1.2* and *1.4* are applied for modeling the translocation process. The main result from these studies show that when the translocation for is controlled by the pore-polymer interaction, the polymer rigidity can enhance translocation, and that triple-peaked translocation time PDF can be obtained when internal pore dynamics are introduced into the model.